Find the derivative of

$y = (x + 2)^2$

$y = (x+2)^3$

$y = (x+2)^5$

The function $y = 6x - 10 = 2(3x - 5)$ is the composite of the functions $y = 2u$ and $u = 3x - 5$. How are the derivatives of these three functions related?

The polynomial $y = 9x^4 + 6x^2 + 1 = (3x^2 + 1)^2$

$y = x^2 = y' = 2x$ $\qquad y' = 36x^3 + 12x$

$$y = (3x^2 + 1)^2$$

$$y' = 2(3x^2 + 1)(6x)$$

$$= 12x(3x^2 + 1)$$

$$= 36x^3 + 12x$$

Section 3.6
The Chain Rule

If f is differentiable at the point y = g(x), and g is differentiable at x, then the composite function $(f \circ g)(x) = f(g(x))$ is differentiable at x, and

$$(f \circ g)'(x) = f'(g(x)) * g'(x)$$

IN ENGLISH

Derivative of the outside * derivative of the inside

Example

$$y = \sqrt{2x + 1}$$

$$y = (2x+1)^{\frac{1}{2}}$$

$$y' = \frac{1}{2}(2x+1)^{-\frac{1}{2}} \cdot 2$$

$$= 1(2x+1)^{-\frac{1}{2}}$$

$$= \frac{1}{\sqrt{2x+1}}$$

$y = \sin(x^2 + x)$

$$y' = \cos(x^2 + x)(2x + 1)$$

Not

$$y' = 2x + \cos(x^2 + x)$$

Right

$$y' = (2x + 1)\cos(x^2 + x)$$

$y = \sin(x^2 + 4)$

$y' = 2x\cos(x^2+4)$

$$y = \sqrt{4x^2 + 2}$$

$$y = (4x^2+2)^{\frac{1}{2}}$$

$$y' = \frac{1}{2}(4x^2+2)^{-\frac{1}{2}}(8x)$$

$$= 4x(4x^2+2)^{-\frac{1}{2}}$$

$$= \frac{4x}{\sqrt{4x^2+2}}$$

$y = \tan \sqrt[3]{x}$

$y = \tan^3 \sqrt{x}$

$y = (\tan \sqrt{x})^3$

$y' = (3\tan^2 \sqrt{x})(\sec^2 \sqrt{x})\left(\frac{1}{2} x^{-\frac{1}{2}}\right)$

$= \frac{3 \tan^2 \sqrt{x} \sec^2 \sqrt{x}}{2\sqrt{x}}$

g(t) = tan (5 - sin 2t)

$$g'(t) = \sec^2(5 - \sin 2t)(-\cos 2t)(2)$$

$$= -2\sec^2(5 - \sin 2t)(\cos 2t)$$

$$\frac{d}{dx}(\tan^2 x)$$

$$y' = 2\tan x \sec^2 x$$

Find y'

$y = \sin^2(3x - 4)$

$y = (\sin(3x-4))^2$

$y' = 2\sin(3x-4) \cdot \cos(3x-4) \cdot 3$

$y' = 6\sin(3x-4)\cos(3x-4)$

Find y'

$$-(x^4 + 3x)^{-2} (4x^3 + 3)$$

Find y'

$4\cos^3 x * (-\sin x) - 4x$

$$y' = 12\cos^2 x \cdot (-\sin x)(-\sin x) + 4\cos^3 x\,(-\cos x) - 4$$

$$= 12\cos^2 x \sin^2 x - 4\cos^4 x - 4$$

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